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VORTICITY–VELOCITY FORMULATION IN THE COMPUTATION OF FLOWS IN MULTICONNECTED DOMAINS

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SUMMARY

The thermofluid dynamic fields in two-dimensional multiconnected domains are analysed by solving the Navier–Stokes equations with the Boussinesq approximation in the vorticity–velocity formulation. The need of an integral condition for the pressure to be single-valued on each independent irreducible loop, in analogy with the ω–ψ formulation, is demonstrated. The field equations are discretized by a finite difference technique and solved at the steady state via an alternating direction implicit method of a scalar type. Two test cases at low Reynolds and Rayleigh numbers are considered: the multiconnected driven cavity and an annulus with isothermal walls and stationary or rotating inner cylinder.

KEY WORDS Navier–Stokes equations Vorticity–velocity Multiconnected domains Pressure single-valuedness

1. INTRODUCTION

Fluid flow and heat transfer in multiconnected domains have received considerable attention by many researchers because of their important applications in physics and engineering. Some technological applications are the journal bearings of rotating machinery and shafts, rod-bundle systems of heat exchangers and of liquid metal boiling reactors, double aerofoils, etc.

In this paper we present a numerical method based on the vorticity–velocity formulation of the Navier–Stokes equations[1,2] to study steady flows in multiconnected regions.

This formulation has distinct advantages when applied to some categories of problems. First, the ω–$\mathbf{u}$ form is simpler than the primitive variable form ($\mathbf{u}$, p) since the pressure does not appear explicitly in the field equations and thus the well known difficulty connected with the determination of the pressure boundary value in incompressible flows is avoided.[3] Furthermore, as demonstrated by Speziale,[4] the ω–$\mathbf{u}$ form has a striking advantage when applied to problems in a non-inertial frame of reference because the non-inertial effects only enter into the solution of the problem through the implementation of boundary conditions. On the other hand, the primitive variable formulation is more versatile, in particular when studying three-dimensional flows. On comparing the proposed ω–$\mathbf{u}$ form and the standard vorticity–streamfunction (ω–ψ) form, only the kinematic aspect has to be considered because the dynamical aspect is governed by the vorticity transport equation in both formulations. The adoption of the streamfunction ψ determines the solenoidal velocity field unequivocally. Although the proposed formulation requires the solution of two Poisson equations for the velocity components, and the continuity

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equation is solved only implicitly, the use of the physical variables **u** and the implementation of natural boundary conditions make this formulation simpler than the vorticity–vector potential formulation when used in the solution of three-dimensional problems.[5,6]

Particular attention has to be paid to the condition of pressure single-valuedness in multiconnected domains. In fact, this condition is not implicitly imposed in the formulations in terms of vorticity since the pressure is eliminated by applying the curl operator to the momentum equation. Many authors solve the Navier–Stokes equations in terms of ω–ψ variables in multiconnected regions by imposing an integral condition for the streamfunction evaluation on the surface of internal bodies. That is, they have one more unknown (flow rate in individual channels) and one more equation of integral type (pressure single-valuedness on irreducible circuit) for each connection. In particular, Launder and Ying[7] study the laminar flow between rotating isothermal eccentric cylinders in a bipolar co-ordinate system, Thompson *et al.*[8] discuss the difficulties of using the ω–ψ formulation rather than the primitive variable formulation in the computation of flow around a double aerofoil, Lee[9] investigates the natural and mixed convection between concentric and eccentric heated rotating cylinders, and Adlam[10] solves natural convection problems in a cavity where there are internal bodies.

On the other hand, an integral condition for the pressure single-valuedness could appear to be unnecessary when using the ω–**u** formulation because this form requires boundary conditions connected only with the velocity, so that the flow rate in the individual channels, which is also unknown, does not appear explicitly in the field equations as it does in the ω–ψ formulation. This explains why in the papers available in the literature this condition is not enforced.[11,12] In this paper we demonstrate by theoretical arguments, analytical solutions and numerical experiments that the ω–**u** form, in analogy with the ω–ψ form, requires the enforcement of an integral condition for the pressure to be single-valued when solving multiconnected problems.

The numerical techniques employed in the present study do not differ very much from those explained in Reference 1 and may be summarized in the following steps.

(1) The variables are located on a staggered grid to satisfy the continuity equation on individual cells better.
(2) The equations are discretized by second-order accurate central differences on a uniform mesh.
(3) The conservative form is adopted for the vorticity transport equation in order to verify the conservation of mean vorticity.
(4) The advective form is adopted for the energy balance in order to discretize the convective term close to the boundary correctly.
(5) The pressure single-valuedness condition, explicitly imposed in terms of the vorticity derivative on each internal boundary, is discretized by a second-order forward derivative.
(6) All the equations are parabolized in time so that they are solved exactly at the steady state only.
(7) An alternating direction implicit (ADI) method of a scalar type is used to integrate in time the four equations governing ω, u, v and θ.

A driven cavity with a hole in the centre is used as a test case, and very good agreement with the numerical results of Reference 13 and with analytical solutions is obtained. Finally, the thermofluid dynamic fields in a two-dimensional annulus with differently heated walls and with a stationary or rotating inner cylinder are studied. The present results for this problem are compared with those of Reference 11, finding, as expected, a significant difference, and with a solution obtained by the primitive variable formulation (**u**, p) of the Navier–Stokes equation,[14] finding identical results.

2. FORMULATION OF THE FLUID DYNAMICS EQUATIONS IN TERMS OF VORTICITY–VELOCITY

The fluid dynamics equations for the planar laminar steady flow of an incompressible fluid in the vorticity–velocity formulation with the Boussinesq approximation are

$$(\mathbf{u}\cdot\nabla)\boldsymbol{\omega}=\frac{1}{Re}\nabla^2\boldsymbol{\omega}+\frac{Gr}{Re^2}\nabla\times(\theta\mathbf{j}), \tag{1}$$

$$\nabla^2\mathbf{u}=-\nabla\times\boldsymbol{\omega}, \tag{2}$$

$$(\mathbf{u}\cdot\nabla)\theta=\frac{1}{RePr}\nabla^2\theta. \tag{3}$$

Here Re, Gr and Pr are the Reynolds, Grashof and Prandtl numbers respectively, defined as

$$Re=u'L/\nu, \qquad Gr=\beta\Delta TL^3g/\nu^2, \qquad Pr=\rho C\nu/k,$$

where u' is the reference velocity, L is the reference length (defined later), ν is the kinematic viscosity, β is the coefficient of thermal expansion, ΔT is the reference temperature interval, g is the modulus of gravitational acceleration, ρ is the density, C is the specific heat and k is the thermal conductivity. The vorticity $\boldsymbol{\omega}=(0, 0, \omega)$ is defined as usual:

$$\boldsymbol{\omega}=\nabla\times\mathbf{u}. \tag{4}$$

3. TEST CASES

In order to study the accuracy and stability of the ω–$\mathbf{u}$ formulation in multiconnected problems of different geometry and boundary conditions, two test cases are considered and analysed: a square driven cavity (side L) with a hole in the centre and a two-dimensional annulus (gap L) with isothermal walls at different temperatures and with a stationary or rotating inner cylinder. Therefore the governing equations (1)–(3) are to be written in Cartesian and polar co-ordinate systems.

For the forced convection problem of the multiconnected driven cavity, the parabolized version of the governing equations in Cartesian co-ordinates is

$$\omega_t+(u\omega)_x+(v\omega)_y-\frac{1}{Re}\nabla^2\omega=0, \tag{5}$$

$$Re\, u_t-\nabla^2u-\omega_y=0, \tag{6}$$

$$Re\, v_t-\nabla^2v+\omega_x=0. \tag{7}$$

Here the standard conservative form is adopted for the convective terms in order to conserve exactly the mean vorticity as required by the Stokes theorem.[1]

The boundary conditions for the problem of the driven multiconnected cavity are sketched in Figure 1. In particular, the zero-slip and impermeability conditions are enforced at all solid walls for velocity equations (6) and (7). The vorticity definition

$$\omega=v_x-u_y \tag{8}$$

is used as the boundary condition for equation (5). Because of the multiple connection of the solution domain, the system of equations requires a pressure single-valuedness constraint as described in the next section.

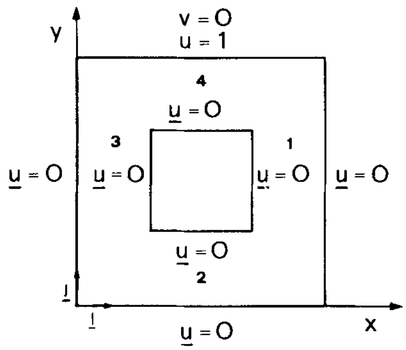

Figure 1. Sketch of the driven cavity with a hole in the centre

The parabolized version of the governing equations in polar co-ordinates is

$$r\omega_t + (ru\omega)_r + (v\omega)_\phi - \frac{1}{Re}\left[(r\omega_r)_r + \frac{1}{r}\omega_{\phi\phi}\right] - \frac{Gr}{Re^2}(\theta_\phi \sin\phi - r\theta_r \cos\phi) = 0, \tag{9}$$

$$r\,Re\,u_t - [r(ru)_r]_r - u_{\phi\phi} - r\omega_\phi = 0, \tag{10}$$

$$r^2\,Re\,v_t - r^2\left[\frac{1}{r}(rv)_r\right]_r - v_{\phi\phi} - 2u_\phi + r^2\omega_r = 0, \tag{11}$$

$$r\theta_t + ru\theta_r + v\theta_\phi - \frac{1}{Re\,Pr}\left[(r\theta_r)_r + \frac{1}{r}\theta_{\phi\phi}\right] = 0, \tag{12}$$

where the co-ordinate system is defined in Figure 2 together with the boundary conditions for the velocity and energy equations (10)–(12). The vorticity definition

$$\omega = \frac{1}{r}(rv)_r - \frac{1}{r}u_\phi \tag{13}$$

is used as the boundary condition for equation (9). In analogy with the Cartesian case, an integral constraint has to be enforced for the pressure single-valuedness.

The form of equations (10) and (11) is essential for mass conservation, which is not explicitly imposed in the present formulation, as explained in Section 4.

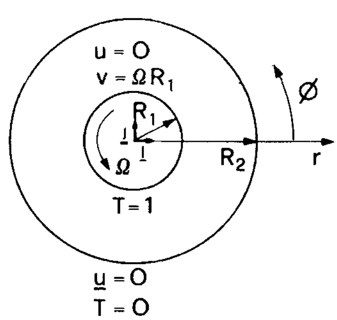

Figure 2. Sketch of the two-dimensional annulus

4. PRESSURE SINGLE-VALUEDNESS IN MULTICONNECTED DOMAINS

It is well known that the formulation in terms of the primitive variables ($\mathbf{u}$, p) is more complex than the formulations in terms of the derived variables ω–ψ and ω–$\mathbf{u}$, because the pressure appears in the field equations. Nevertheless, since the pressure is a dependent variable, it does statisfy implicitly the single-valuedness condition

$$\oint_l \nabla P \cdot \mathrm{d}\mathbf{l} = 0 \tag{14}$$

in both simple and multiconnected domains. On the other hand, in the formulations in terms of vorticity, ω–ψ and ω–$\mathbf{u}$, condition (14) cannot be satisfied implicitly because the pressure gradient has been eliminated by applying the curl operator to the momentum equation. Consequently, two different situations may occur.

(i) Condition (14) is identically satisfied in simply connected problems by the effect of the Stokes theorem:

$$\oint_l \nabla P \cdot \mathrm{d}\mathbf{l} = \iint_S \nabla \times \nabla P \cdot \mathbf{n}\, \mathrm{d}S = 0. \tag{15}$$

(ii) Condition (14) must be explicitly enforced in multiconnected problems in order for pressure to be single-valued, because the Stokes theorem ensures the pressure single-valuedness on reducible curves only (local integrability).

Many authors who analyse the flows in multiconnected domains using the ω–ψ formulation[7–10] impose the integral equation (14) to evaluate the streamfunction on the surface of the inner bodies. On the other hand, an integral condition for the pressure single-valuedness could appear to be unnecessary when using the ω–$\mathbf{u}$ formulation because this formulation requires boundary conditions connected only with the velocity, so that the flow rate in the individual channels, which is also unknown, does not appear explicitly in the field equations as it does in the ω–ψ formulation. This explains why condition (14) is erroneously not imposed by Farouk and Fusegi[11,12] in solving a buoyancy problem in a two-dimensional annulus in terms of ω–$\mathbf{u}$ variables.

To demonstrate that condition (14) is also required when using the ω–$\mathbf{u}$ formulation, let us consider the flow between two concentric isothermal ($Gr=0$) cylinders. The Navier–Stokes equations (9)–(13) in polar co-ordinates, upon elimination of the azimuthal derivative, the radial velocity and the temperature, become

$$(r\omega_{,r})_{,r} = 0, \tag{16}$$

$$-\left[\frac{1}{r}(rv)_{,r}\right]_{,r} + \omega_{,r} = 0, \tag{17}$$

$$\omega = \frac{1}{r}(rv)_{,r}. \tag{18}$$

The boundary conditions are $v(R_1)=1$ and $v(R_2)=0$. The solution of this differential problem is

$$v(r) = (A + \tfrac{1}{2}BD)r + \left(\frac{1}{2}CD - \frac{1}{\gamma^2}AR_1^2\right)\frac{1}{r} + D\frac{r}{2}\ln r, \tag{19}$$

$$\omega = 2A + BD + \frac{D}{2} + D\ln r, \tag{20}$$

where

$$A=\frac{1}{R_1}\frac{1}{1-1/\gamma^2},\qquad B=\frac{\ln(R_1/\gamma)-\gamma^2\ln R_1}{\gamma^2-1},\qquad C=R_1^2\frac{\ln\gamma}{\gamma^2-1},\qquad \gamma=\frac{R_1}{R_2}. \tag{21}$$

The solution (19), (20) represents a family of functions depending on one parameter (D). All these functions identically satisfy the differential equations (16) and (17), the vorticity definition (18) and the boundary conditions. The unknown parameter D must be determined by (14).

Let us consider the momentum equations in terms of velocity,

$$\nabla P=-(\mathbf{u}\cdot\nabla)\mathbf{u}+\frac{1}{Re}\nabla^2\mathbf{u}+\frac{Gr}{Re^2}\theta\mathbf{j}, \tag{22}$$

and let us enforce condition (14) along an arbitrary circuit l looping the inner body once:

$$\oint_l \nabla P\cdot d\mathbf{l}=-\oint_l(\mathbf{u}\cdot\nabla)\mathbf{u}\cdot d\mathbf{l}+\frac{1}{Re}\oint_l\nabla^2\mathbf{u}\cdot d\mathbf{l}+\frac{Gr}{Re^2}\oint_l\theta\mathbf{j}\cdot d\mathbf{l}=0. \tag{23}$$

If we consider as the circuit the outer cylinder, which is isothermal and stationary, equation (23) reduces to

$$\oint_l \omega_r\, dl=0 \tag{24}$$

The integral condition (24) holds for the general problems described in Figures 1 and 2, while in the case of the one-dimensional problem described in this section, it becomes of local type:

$$\omega_r=0. \tag{25}$$

The value of constant $D=0$, which gives the well known analytical solution (Reference 15, p. 201), is obtained by means of equation (25).

The same argument holds in Cartesian co-ordinates for solving a simple Couette flow with periodic boundary conditions.

5. NUMERICAL MODEL

The numerical model is obtained by discretizing via a finite difference technique the governing equations of Section 3. A second-order centred scheme on a regular mesh is adopted for both first and second derivatives, and a scalar alternating direction implicit method is used for the false time integration.

Particular attention has to be paid to the variable location in the computational molecule in order to satisfy mass conservation, which is not identically satisfied as it is in the streamfunction formulation. The non-staggered scheme has been used frequently in the ω–$\mathbf{u}$ form of the Navier–Stokes equations[2, 5, 16] because the enforcement of boundary conditions and the evaluation of convective terms are straightforward. The staggered scheme[1, 17, 18] is not very easy to implement in the discretized form, but it ensures that the continuity equation is identically satisfied in the numerical procedure, as is explained in the following.

Let us consider the velocity components yielded by the discrete derivative of the streamfunction, and substitute them in the continuity equation written on the computational cell. In the case of staggered velocities, the continuity equation is identically satisfied. On the other hand, a residual is produced if non-staggered velocities are used. This residual of the continuity equation may spoil the accuracy of the solution, as may be shown by solving the two-dimensional Poiseuille flow between two parallel walls in a square domain. We find, using 8×8 cells and a first-order vorticity boundary condition, perfect mass conservation in the case of the staggered scheme and a loss of mass ($\sim 12\%$) in the case of the non-staggered scheme.

In relation to the discussion of the previous section, the algebraic equation system corresponding to the discretized form of the governing equations and the boundary conditions have to be singular. In fact, it is possible to demonstrate in the monodimensional case by Gauss elimination that one boundary condition on ω is linearly dependent on the other equations. Therefore the vorticity definition on one mesh point of the frontier of each internal body has to be substituted by the discretized form of the integral condition (24), giving a well conditioned algebraic equation system.

The numerical implementation of this integral condition is not straightforward; in fact, the vorticity derivative normal to the boundary has to be computed at least by a second-order accurate, one-sided derivation formula. Furthermore, the unknown vorticity in the considered point is correlated, via the integral relation, with the vorticity derivative in all the other points of the same boundary. Therefore this condition must be computed explicitly, with a lowering of convergence speed, otherwise the three-diagonality of the algebraic equation system is lost.

6. RESULTS AND DISCUSSION

The square driven cavity (side L) with a hole in the centre (side L_1), sketched in Figure 1, is analysed first. The Reynolds number, based on the cavity side, is firstly held fixed at $Re = 1$ to achieve a comparison with Reference 13. A mesh of up to 81×81 points is adopted for the computations. The symmetric problem ($L_1/L = 1/5$) described in Figure 3 is solved initially to ensure that the integral condition (24) does not introduce a spurious circulation along a circuit looping the inner body once. The streamlines show perfect symmetry. In Figure 4 the flow induced by the horizontal movement of the top wall is shown. Different ratios between the sides of the hole and the cavity are considered: $L_1/L = 1/10, 1/5, 1/3, 3/5, 4/5$. Note that the structure of the flow for $L_1/L \leqslant 1/5$ (Figure 4(a)) is similar to that of a driven cavity, but the no-slip condition on the inner body causes a decrease of velocity, so that the maximum value of the streamfunction is smaller in the present case. The upper recirculating zone is still present for $L_1/L \geqslant 4/5$ (Figure 4(c)), and we find the surprising result that the structure of the flow does not change significantly for the gap going to zero ($L_1/L \rightarrow 1$). This may be tentatively explained by considering that the domain comprises four straight channels joined by small corner areas. In these channels a type of Couette flow occurs. In particular, in channels 1, 2 and 3 a pressure gradient (constant except for the effects

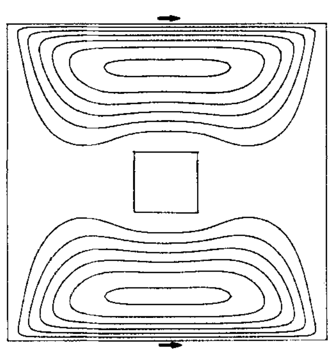

Figure 3. Driven cavity with a hole in the centre: symmetric problem; $Re = 1$

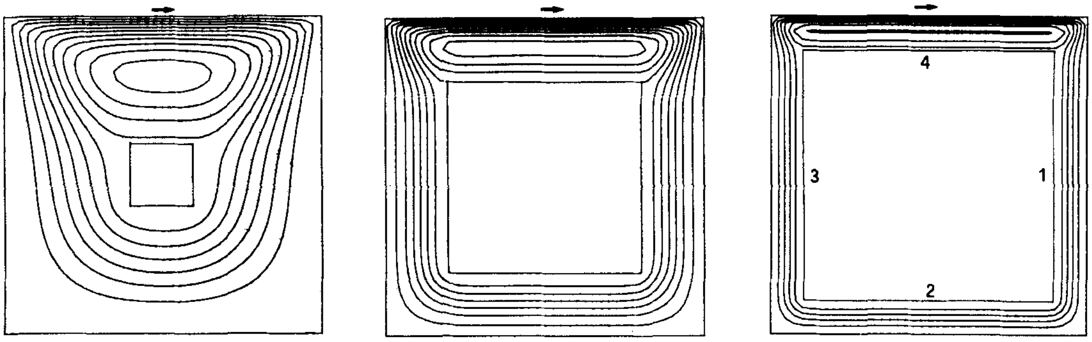

Figure 4. Driven cavity with a hole in the centre at $Re=1$: (a) $L_1/L=1/5$; (b) $L_1/L=3/5$; (c) $L_1/L=4/5$

of the corners) is the only forcing term, so that the pressure must decrease in the flow direction. Consequently, in channel 4 the pressure must increase in the flow direction, and a general case of Couette flow occurs by the superposition of the effect of the adverse pressure gradient, which gives a parabolic profile, and that of the driving action of the moving top wall, which gives a simple shear flow. An analytical solution can be found by considering the asymptotic problem for $L_1/L \rightarrow 1$ (gap $\rightarrow$ zero) and imposing the compatibility conditions on flow rate and pressure drop in the various channels. In Figure 5 the numerical flow rate and maximum reversal velocity in channel 4 are compared with those obtained analytically for the asymptotic problem. Two different Reynolds numbers are considered, $Re=1$ and $Re=200$. In the former case very good agreement is found not only for the small gap ($L_1/L=4/5$) but also for the other geometrical configurations. In the latter case the numerical and analytical solutions are in good agreement only for the small gap but show, as expected, a difference which increases with increasing gap. The velocity profiles in the middle section of channels 4 and 2 for $L_1/L=1/3$ are shown in Figure 6; the present results are compared with those of Reference 13 and show fairly good agreement.

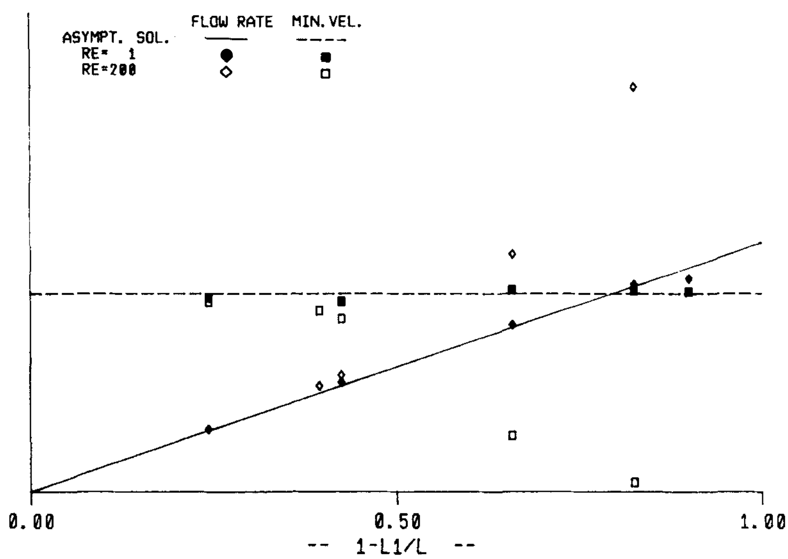

Figure 5. Driven cavity with a hole: comparison with asymptotic solution

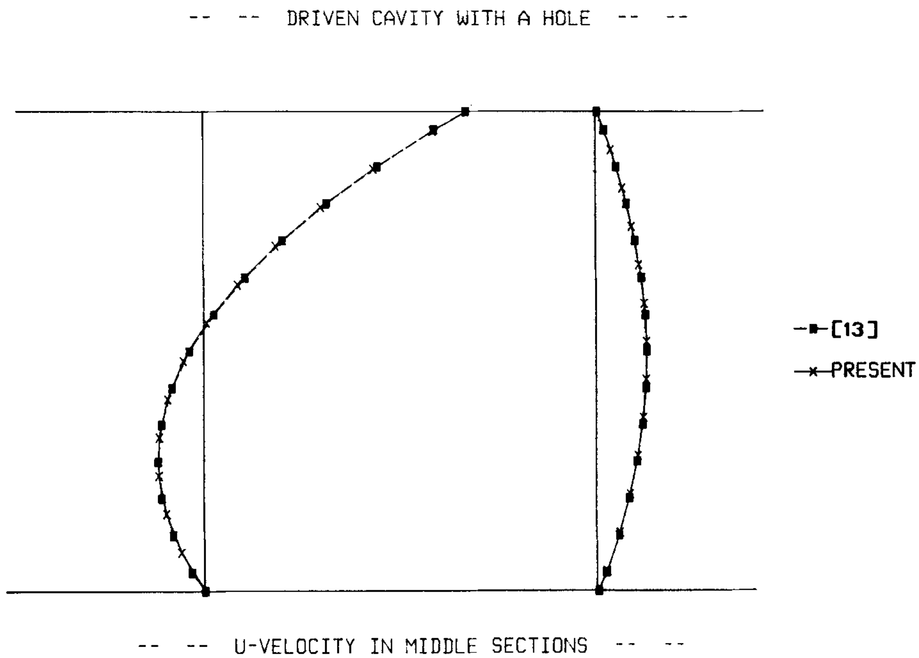

Figure 6. Driven cavity with a hole in the centre: velocity profile in the middle section at $Re=1$; comparison with Reference 13

The two-dimensional annulus between concentric cylinders, sketched in Figure 2, is considered next. The ratio of the gap width L to the inner cylinder radius (R_1) is held fixed at 1·6, the same as in Reference 19. The isothermal axisymmetric problem described in Section 4 is considered first to verify the accuracy of the numerical model in comparison with the analytical solution ((19) and (20)). In this very simple case we found the following results, which substantiate the statements of the previous sections.

(i) The discretized form of the one-dimensional problem ((16) and (17)), using the velocity boundary condition and vorticity definition on both sides, gives a singular coefficient matrix according to the analytical considerations. In contrast, a second-order accurate solution is found using condition (25).
(ii) The discretized form of the two-dimensional problem ((9)–(11)) gives a solution whose flow rate depends on the initial guess if velocity boundary conditions and definition (13) of vorticity on the wall of both cylinders are imposed. But a solution in agreement with the analytical one is found if the integral condition (24) is used to compute the vorticity boundary condition on a point of the boundary. The numerical solution is, as expected, second-order accurate and solenoidal to a round-off error (10^{-12}).

Natural and mixed convention in the annulus of Figure 2 is considered next. The direction of gravitational force is $\phi=-90°$. The Rayleigh number is defined as $Ra=Pr\times Gr$.

First we consider the natural convection problem which was experimentally investigated by Kuehn and Goldstein.[19] Owing to the symmetry, we solve only one half of the annulus and the integral condition (24) is not enforced. A typical mesh of 81×41 is used in the computation. The streamlines and isotherms for $Ra=5\times10^4$ and $Pr=0{\cdot}706$ are shown in Figure 7; they are in qualitative agreement with those of Reference 19.

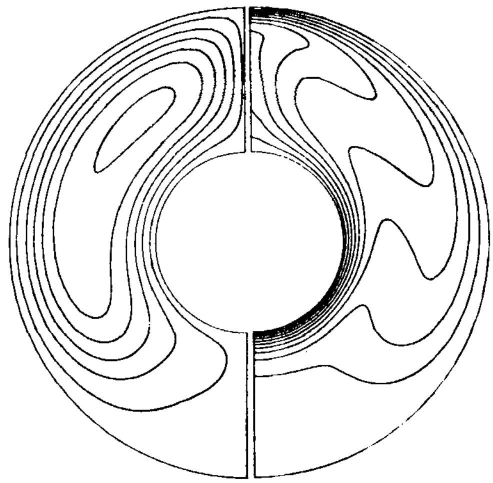

Figure 7. Two-dimensional annulus: streamlines and isotherms; $Ra = 5 \times 10^4$, $Pr = 0{\cdot}706$

In Figure 8 the local equivalent conductivity K_{eq}, defined as the actual heat flux divided by the heat flux that would occur by pure conduction, is shown in comparison with the experimental measurements of Reference 19. The disagreement between the two results is less than 10%.

Finally, the mixed convection problem proposed by Farouk and Fusegi[11] is studied; in this case Ra is held fixed at 10^3, the Prandtl number is set at a constant value of 0·72, and the Reynolds number based on the annulus gap and azimuthal velocity of the inner cylinder is varied in the range 0–37·3, the same as in Reference 11.

The results are shown as a function of the non-dimensional group $\sigma = Gr/Re^2$, which represents the relative strength of the convective and inertial effects.

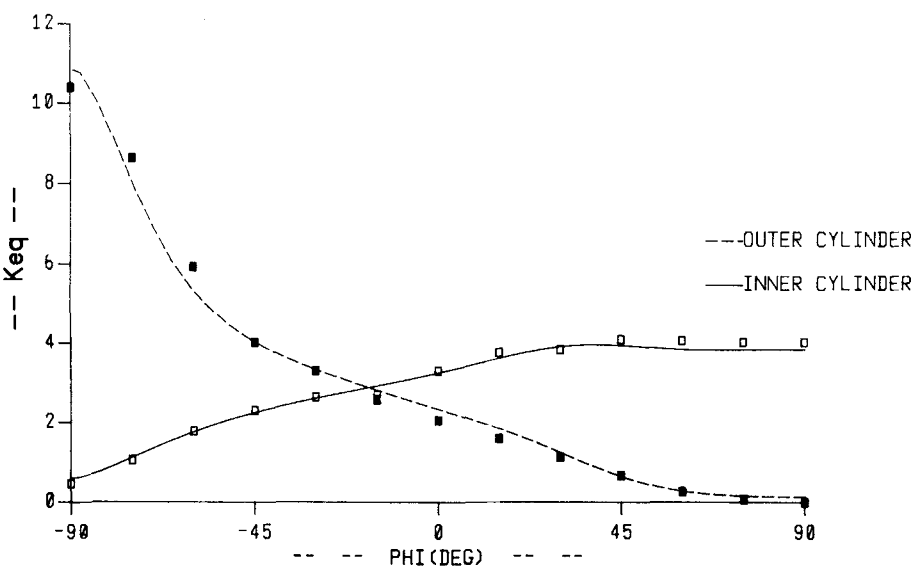

Figure 8. Two-dimensional annulus: comparison with Reference 19

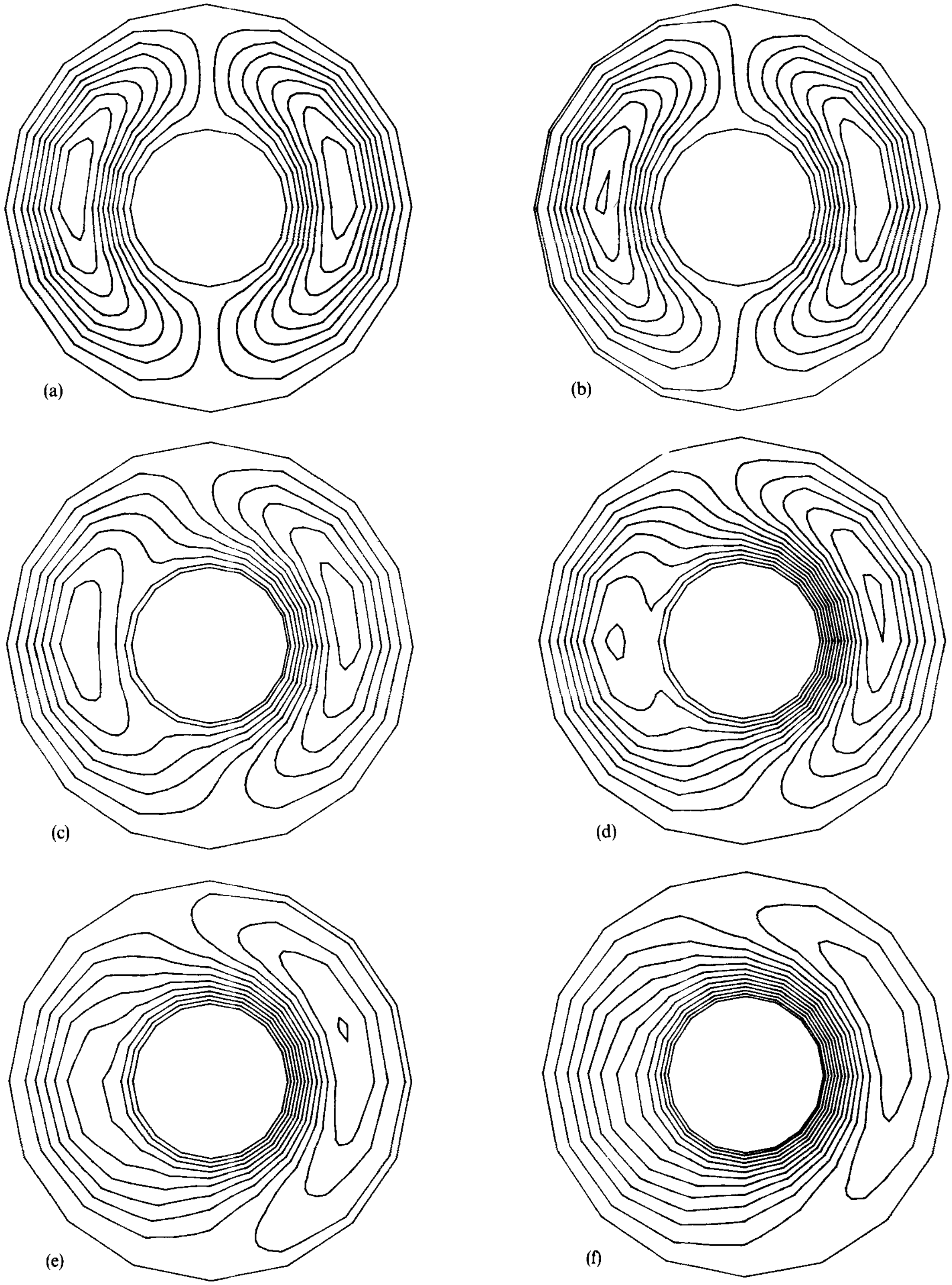

Figure 9. Two-dimensional annulus: streamlines; $Ra = 10^3$, $\sigma = \infty$, 1389, 10, 5, 2, 1

Figure 9 displays the streamlines for σ varying between ∞ and 1. Note that for small σ values, owing to the combined effect of the natural convection and the driving of the rotating cylinder, the streamlines tend to pass near the inner cylinder on the right side and near the outer cylinder on the left side of the annulus. Furthermore, for $\sigma \geqslant 5$ two stagnation points are present on the left side. These points disappear for smaller σ values because the inertial forces prevail over the buoyancy. Thermal isolines are not presented, because at the low Peclet number studied here ($Pe = Pr \times Re$), the effect of advection on the temperature field is negligible. The azimuthal velocity profiles along the gap at $\phi = 0°$ and $\phi = 180°$ are shown in Figure 10. The profiles described above are compared with those reported in Reference 11 (Figure 11). Good agreement between the present results and those of Reference 11 is seen for the natural convection problem ($\sigma = \infty$). On the other hand, a discrepancy of about 40% in the flow rate is seen between the two results for $\sigma = 1$. To find a reference solution in this case ($\sigma = 1$), the problem is solved using the primitive variable formulation ($\mathbf{u}$, p) which does not require the condition of pressure single-valuedness because the pressure is an independent variable.[14] The present results are in very good quantitative agreement with the reference solution but not with the velocity of Reference 11. In our opinion this discrepancy has to be attributed to the violation of the condition of global integrability of the pressure in Reference 11 rather than to the difference in mesh resolution (Figure 12) at the low *Ra* (1000) and *Re* (37·3) numbers considered, as confirmed also by the good agreement displayed for $\sigma = \infty$ (symmetric solution).

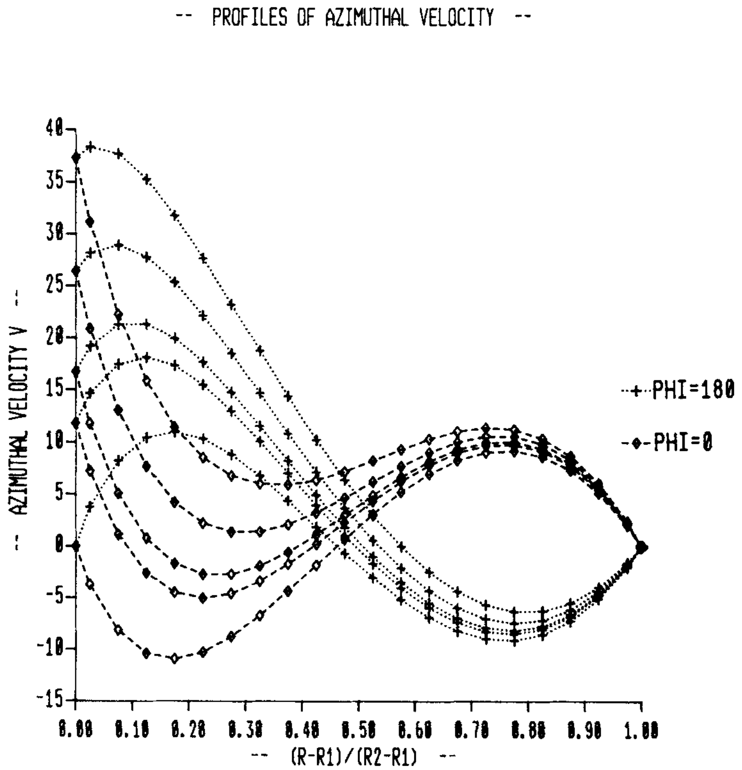

Figure 10. Two-dimensional annulus: azimuthal velocity profiles for $\phi = 0°$ and $\phi = 180°$

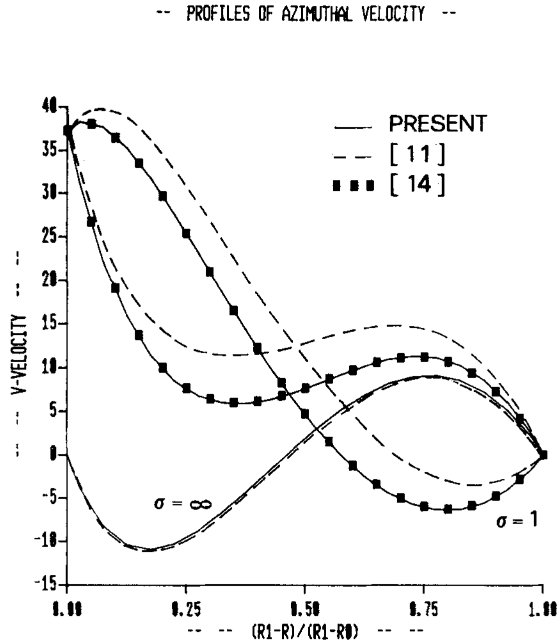

Figure 11. Two-dimensional annulus: azimuthal velocity profiles for $\phi = 0°$ and $\phi = 180°$; comparison with References 11 and 14

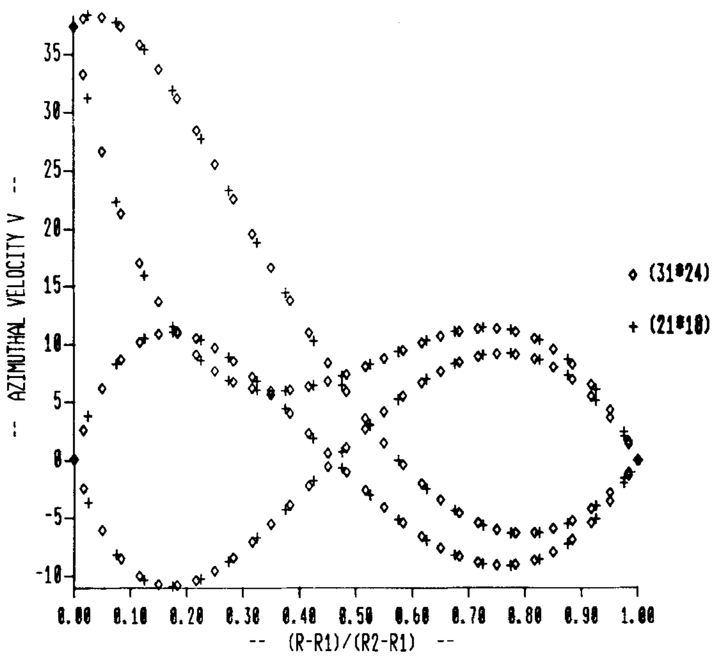

Figure 12. Two-dimensional annulus: mesh dependence; $\sigma = \infty$, 1

7. CONCLUSIONS

A multiconnected driven cavity and natural and mixed convection in a two-dimensional annulus are studied by a numerical model based on the ω–$\mathbf{u}$ formulation of the Navier–Stokes equations. We demonstrate by theoretical arguments that the use of the physical variable ω does not guarantee pressure single-valuedness in multiconnected domains. Therefore a constraint of the integral type has to be enforced on each internal body in order to avoid spurious mass flow in the single channels. The accuracy of the present numerical solutions is tested by comparison with analytical, numerical and experimental results.

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